# Quartic Force Field Training 

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Fortenberry Patch

What is a quartic force field?

## What is a quartic force field?

## Definition

A vector field that describes a force acting on a particle at various positions in space

(a) A gravitational force field

(b) A vector field

## What is a quartic force field?

The obvious

$$
F=m a
$$

## What is a quartic force field?

The obvious

$$
F=m a
$$

The more useful

$$
F=-\nabla V=-\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)
$$

## How does this relate to us?



Figure: Potential vs displacement for a 1-D harmonic oscillator

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Suppose we do not know the potential energy function for a molecule. How could we approximate it?


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Use a Taylor series!

## How does this relate to us?

Taylor series

## Definition

An infinite sum of terms that are expressed in terms of the function's derivatives at a single point, a

$$
f(x)_{a} \approx f(a)+\frac{f^{\prime}(a)}{1!}(x-a)+\frac{f^{\prime \prime}(a)}{2!}(x-a)^{2}+\ldots
$$

Example

$$
e_{a=0}^{x} \approx \frac{e^{0}}{0!}+\frac{e^{0}}{1!}(x-0)+\frac{e^{0}}{2!}(x-0)^{2}+\ldots=1+x+\frac{x^{2}}{2}+\ldots
$$

## What is a quartic force field?

You've probably seen this in all of our papers:

## Definition

A quartic force field is a fourth-order Taylor series expansion of the potential energy portion of the internuclear Watson Hamiltonian

And this in some of them:

## Equation

$$
V=\frac{1}{2} \sum_{i j} F_{i j} \Delta_{i} \Delta_{j}+\frac{1}{6} \sum_{i j k} F_{i j k} \Delta_{i} \Delta_{j} \Delta_{k}+\frac{1}{24} \sum_{i j k l} F_{i j k l} \Delta_{i} \Delta_{j} \Delta_{k} \Delta_{l}
$$

## Where did Taylor go?

Let's expand the potential energy for the 1-D harmonic oscillator, $V(r)$, about its equilibrium displacement $r_{\text {eq }}$, and let this be equal to zero.

$$
\begin{aligned}
V(r)_{r_{e q}} \approx \frac{1}{0!} V\left(r_{e q}\right) & +\frac{1}{1!}\left(\frac{d V}{d r}\right)_{r_{e q}}\left(r-r_{e q}\right)+\frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2} \\
& +\frac{1}{3!}\left(\frac{d^{3} V}{d r^{3}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{3}+\frac{1}{4!}\left(\frac{d^{4} V}{d r^{4}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{4}
\end{aligned}
$$

What do we know about the first two of these terms?

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+ & \frac{1}{3!}\left(\frac{d^{3} V}{d r^{3}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{3}+\frac{1}{4!}\left(\frac{d^{4} V}{d r^{4}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{4}
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\end{array}+\frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2}\right)
$$

What do we know about the first two of these terms?

- The first cancels because we defined $r_{e q}$ as our zero of potential
- The second because the derivative at the minimum is zero


## Where did Taylor go?

This leaves us with

$$
\begin{aligned}
& V(r)_{r_{e q}} \approx \frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2} \\
&+\frac{1}{3!}\left(\frac{d^{3} V}{d r^{3}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{3} \\
&+\frac{1}{4!}\left(\frac{d^{4} V}{d r^{4}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{4}
\end{aligned}
$$

which should be looking vaguely familiar

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which should be looking vaguely familiar

$$
V=\frac{1}{2} \sum_{i j} F_{i j} \Delta_{i} \Delta_{j}+\frac{1}{6} \sum_{i j k} F_{i j k} \Delta_{i} \Delta_{j} \Delta_{k}+\frac{1}{24} \sum_{i j k l} F_{i j k l} \Delta_{i} \Delta_{j} \Delta_{k} \Delta_{l}
$$

## Where did Taylor go?

Let's zoom in on the first term. Looking familiar?

## Comparison

$$
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& V(r)_{r_{e q}} \approx \frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2} \\
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\end{aligned}
$$

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Let's zoom in on the first term. Looking familiar?

## Comparison

$$
\begin{aligned}
& V(r)_{r_{e q}} \approx \frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2} \\
& V=\frac{1}{2} \sum_{i j} F_{i j} \Delta_{i} \Delta_{j} \\
& V=\frac{1}{2} k x^{2}
\end{aligned}
$$

The first term is just the harmonic oscillator potential energy, accounting for the fact a second derivative can be two steps in one direction (like $x^{2}$ ), or one step in different directions (like $\Delta_{i} \Delta_{j}$ )

## What is a quartic forec field?

Quartic just refers to the fourth-degree part of the Taylor series


The quadratic or harmonic terms give us the red curve, while the third- and fourth-order terms treat the anharmonicity, giving something like the blue curve.

## How do we make one?

Let's go back to our full quartic equation

$$
V=\frac{1}{2} \sum_{i j} F_{i j} \Delta_{i} \Delta_{j}+\frac{1}{6} \sum_{i j k} F_{i j k} \Delta_{i} \Delta_{j} \Delta_{k}+\frac{1}{24} \sum_{i j k l} F_{i j k l} \Delta_{i} \Delta_{j} \Delta_{k} \Delta_{l}
$$

Of the terms here, we can "easily" get:

- $\mathrm{V} \rightarrow$ comes out of Molpro
- $\Delta \rightarrow$ can be anything we want, typically $0.005 \AA$ or radians

By the way, why the summations?

## How do we make one?

If we were to plot a 1-D case of taking steps $(\Delta)$ and computing the energy with Molpro, we would get something like this


But probably not this smooth

## How do we make one?

To go from the scatterplot to a potential function, we have to

- Fit a function to our points, linear regression


## How do we make one?

To go from the scatterplot to a potential function, we have to

- Fit a function to our points, linear regression
- Use that function to calculate the force constants $\left(F_{i, j, k, l}\right)$

Recall

$$
F=-\nabla V
$$

## How do we make one?

To go from the scatterplot to a potential function, we have to

- Fit a function to our points, linear regression
- Use that function to calculate the force constants $\left(F_{i, j, k, l}\right)$


## Recall

$$
F=-\nabla V
$$

- Convert the force constants into frequencies

Harmonic oscillator again

$$
\omega=\sqrt{\frac{k}{m}}
$$

## How do we make one?

"Fortunately" we have ancient programs that handle these steps for us

- Anpass - fits the function to our points and generates the force constants
- Spectro - determines the frequencies from the force constants
- Intder - the glue that holds all of the steps together



## Getting started

## Geometry optimization

Looking back at these, we see we need to start off at a good minimum

$$
\left.\begin{array}{rl}
V(r)_{r_{e q}} \approx \frac{1}{\theta!} V\left(r_{e q}\right) & +\frac{1}{1!}\left(\frac{d V}{d r}\right)_{r_{e q}}\left(r-r_{e q}\right)
\end{array}+\frac{1}{2!}\left(\frac{d^{2} V}{d r^{2}}\right)_{r_{e q}}\left(r-r_{e q}\right)^{2}\right)
$$



Cancelling the second term, in particular, requires that we are at the minimum energy structure

## Geometry optimization

Assuming Molpro

## CCSD (T)-F12/cc-pVTZ-F12 example, also shortened to F12-TZ

 (with my syntax highlighting "plugin")```
memory, 995, m;
gthresh,energy=1.d-12,zero=1.d-22,oneint=1.d-22,twoint=1.d-22;
gthresh,optgrad=1.d-8,optstep=1.d-8;
nocompress;
geometry={
0
H 1 oh
C 1 oc 2 hoc
N 3 cn 1 ocn 2 180.0
OH=}\quad0.96399073 ANG
OC= 1.30253236 ANG
HOC= 109.54391210 DEG
CN= 1.16094343 ANG
OCN= 176.78899457 DEG
basis=cc-pVTZ-F12
{hf,maxit=500;wf,charge=0, spin=0;accu, 20;}
{ccsd(t)-f12,maxit=250;wf,charge=0,spin=0;orbital,IGNORE ERROR;}
{optg,grms=1.d-8,srms=1.d-8}
```


## Project management (optimizing you)

## What I do

Once that's running, you have a little while to plan your attack

| Molecule | Opt | Freq | Pts | Freqs | Int | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{2} \mathrm{O}$ | $\times$ | $R$ | $R$ |  | $\times$ | Warning in opt.out |
| $\mathrm{H}_{2} \mathrm{~S}$ | R |  |  |  | $x$ |  |
| $\mathrm{~S}_{2} \mathrm{O}$ | R |  |  |  | $x$ |  |

- Opt - Molpro geometry optimization
- Freq - Molpro harmonic frequencies
- Pts - QFF single-point energies
- Freqs - QFF anharmonic frequencies
- Int - intensities, usually done in Gaussian


## Project management

Dependencies/ordering


1 can be done with a script called freqgen.sh, and we'll cover the others now

## Generating points

## intder.in - anatomy



[^0]
## Generating points

## intder.in - anatomy

| FINTDER |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6 | 6 | 0 | 0 | 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |

## Generating points

intder.in - anatomy


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## Generating points

intder.in - anatomy


## Generating points

intder.in - general tips

- Spacing matters! Everywhere
- Don't use tabs
- Don't have trailing whitespace
- Copy a similar line from a known file to compare
- Spacing matters



## Generating points

intder.in - general tips

That looks confusing. What do I actually change?

- Only the Cartesians for now (I think)
- Directives are in the manual if you really need them
- Internal coordinates are our next topic


## Internal coordinates

## Definition

Coordinates with an internal reference, in contrast to Cartesian coordinates, for example, which reference an external set of axes

```
H 1 OH
H 1 OH 2 HOH
OH = 1.0
HOH = 104.5
```

Figure: Z-matrix: (hopefully) familiar internal coordinates


Figure: Cartesian coordinates

## Internal coordinates

Simple internals

## Definition

A single internal coordinate, like a bond stretch, bend, or torsion


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## Internal coordinates

Symmetry internals

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Linear combinations of simple internal coordinates


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Linear combinations of simple internal coordinates

$$
r\left(\mathrm{O}-\mathrm{H}_{1}\right)-r\left(\mathrm{O}-\mathrm{H}_{2}\right)
$$

Anti-symmetric stretch

## Internal coordinates

Symmetry internals

## Definition

Linear combinations of simple internal coordinates
$\angle\left(\mathrm{H}_{1}-\mathrm{O}-\mathrm{H}_{2}\right)$
Bend
$\mathrm{H}_{1}$

$\mathrm{H}_{2}$

## Symmetry

We can characterize a molecule, and its vibrational modes, based on symmetry. Water has $C_{2 v}$ symmetry, which is a pretty common one in our research.

| $\mathbf{C}_{\mathbf{2 v}}$ | $E$ | $\mathrm{C}_{2}(\mathrm{z})$ | $\sigma_{v}(x z)$ | $\sigma_{v}(y z)$ | linear functions, <br> rotations | quadratic <br> functions | cubic <br> functions |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | +1 | +1 | +1 | +1 | $z$ | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ | $\mathrm{z}^{3}, \mathrm{x}^{2} \mathrm{z}, \mathrm{y}^{2} \mathrm{z}$ |
| $\mathrm{A}_{2}$ | +1 | +1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ | xy | xyz |
| $\mathrm{B}_{1}$ | +1 | -1 | +1 | -1 | $\mathrm{x}, \mathrm{R}_{\mathrm{y}}$ | xz | $\mathrm{xz}^{2}, \mathrm{x}^{3}, \mathrm{xy}^{2}$ |
| $\mathrm{~B}_{2}$ | +1 | -1 | -1 | +1 | $\mathrm{y}, \mathrm{R}_{\mathrm{x}}$ | yz | $\mathrm{yz}^{2}, \mathrm{y}^{3}, \mathrm{x}^{2} \mathrm{y}$ |

Figure: Symmetry or character information is found in character tables

## Symmetry

Symmetry operations

- Identity - $E$
- Rotation - $C_{n}$, at left
- Reflection - $\sigma$, at right
- Inversion - $i$, not shown
- Improper rotation - $S_{n}$, not shown
- $S_{1}=\sigma$
- $S_{2}=i$



## Symmetry

- Usually rotations and reflections are enough to assign a symmetry, or when the symmetry is $C_{2 v}$, that's all there is anyway.
- How do we do it?


## Symmetry

- Usually rotations and reflections are enough to assign a symmetry, or when the symmetry is $C_{2 v}$, that's all there is anyway.
- How do we do it? Back to the character table

| $\mathbf{C}_{\mathbf{2 v}}$ | E | $\mathrm{C}_{2}(\mathrm{z})$ | $\sigma_{\mathrm{v}}(\mathrm{xz})$ | $\sigma_{v}(\mathrm{yz})$ | linear functions, <br> rotations | quadratic <br> functions | cubic <br> functions |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | +1 | +1 | +1 | +1 | $z$ | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ | $\mathrm{z}^{3}, \mathrm{x}^{2} \mathrm{z}, \mathrm{y}^{2} \mathrm{z}$ |
| $\mathrm{A}_{2}$ | +1 | +1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ | xy | xyz |
| $\mathrm{B}_{1}$ | +1 | -1 | +1 | -1 | $\mathrm{x}, \mathrm{R}_{\mathrm{y}}$ | xz | $\mathrm{xz}^{2}, \mathrm{x}^{3}, \mathrm{xy}^{2}$ |
| $\mathrm{~B}_{2}$ | +1 | -1 | -1 | +1 | $\mathrm{y}, \mathrm{R}_{\mathrm{x}}$ | yz | $\mathrm{yz}^{2}, \mathrm{y}^{3}, \mathrm{x}^{2} \mathrm{y}$ |

## Symmetry

Determining symmetry

- Perform a symmetry operation
- If the molecule looks the same, assign a 1
- If the molecule looks the "opposite", assign a -1
- Match your $\pm 1$ values to rows of the character table

| $\mathbf{C}_{\mathbf{2 v}}$ | E | $\mathrm{C}_{2}(\mathrm{z})$ | $\sigma_{v}(\mathrm{xz})$ | $\sigma_{v}(\mathrm{yz})$ | inear functions, <br> rotations | quadratic <br> functions | cubic <br> functions |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | +1 | +1 | +1 | +1 | $z$ | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ | $\mathrm{z}^{3}, \mathrm{x}^{2} \mathrm{z}, \mathrm{y}^{2} \mathrm{z}$ |
| $\mathrm{A} \mathrm{A}_{2}$ | +1 | +1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ | xy | xyz |
| $\mathrm{B}_{1}$ | +1 | -1 | +1 | -1 | $\mathrm{x}, \mathrm{R}_{\mathrm{y}}$ | xz | $\mathrm{xz}^{2}, \mathrm{x}^{3}, \mathrm{xy}^{2}$ |
| $\mathrm{~B}_{2}$ | +1 | -1 | -1 | +1 | $\mathrm{y}, \mathrm{R}_{\mathrm{x}}$ | yz | $\mathrm{yz}^{2}, \mathrm{y}^{3}, \mathrm{x}^{2} \mathrm{y}$ |

## Symmetry

## Determining symmetry

What is the symmetry of the anti-symmetric stretch in water? Let the main axis of rotation be the $z$-axis

| $\mathbf{C}_{\mathbf{2 v}}$ | E | $\mathrm{C}_{2}(\mathrm{z})$ | $\sigma_{\mathrm{v}}(\mathrm{xz})$ | $\sigma_{\mathrm{v}(\mathrm{yz})}$ | inear functions, <br> rotations | quadratic <br> functions | cubic <br> functions |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | +1 | +1 | +1 | +1 | $z$ | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ | $\mathrm{z}^{3}, \mathrm{x}^{2} \mathrm{z}, \mathrm{y}^{2} \mathrm{z}$ |
| $\mathrm{A}_{2}$ | +1 | +1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ | xy | xyz |
| $\mathrm{B}_{1}$ | +1 | -1 | +1 | -1 | $\mathrm{x}, \mathrm{R}_{\mathrm{y}}$ | xz | $\mathrm{xz}^{2}, \mathrm{x}^{3}, \mathrm{xy}^{2}$ |
| $\mathrm{~B}_{2}$ | +1 | -1 | -1 | +1 | $\mathrm{y}, \mathrm{R}_{\mathrm{x}}$ | yz | $\mathrm{yz}^{2}, \mathrm{y}^{3}, \mathrm{x}^{2} \mathrm{y}$ |

4 $\mathrm{r}\left(\mathrm{O}-\mathrm{H}_{1}\right)-\mathrm{r}\left(\mathrm{O}-\mathrm{H}_{2}\right)$
Anti-symmetric stretch


## Symmetry <br> Why do we need it?

- Limits the number of displacements we have to calculate
- Part of what determines the IR activity of a mode
- It goes in your computational details


## Generating points

## intder.in - water



Figure: Not water

## Generating points

## intder.in - water



Figure: Water

## Generating points

## Procedure

- Copy (yank) optimized geometry (in Bohr) from the Molpro log file
- Paste it into the intder.in file and make sure alignment is perfect
- Run \$ intder < intder.in > intder.out or something similar
- Use pts-gen.py to generate the single-point energy calculations from intder output
- \$ pts-gen.py h2o O H H
in our water example
- \$ cd inp \&\& sh submit to submit the calculations


## Gathering output

- Grab all the energies from output files
- \$ grep "CCSD(T)-F12b" *.out > energy.dat or use check.sh
- Check that the number of energies is the same as the number of displacements
- handy command for this is \$ wc -l filename "word count -lines"
- \$ comp.py h2o
to print missing ones to terminal
- \$ comp.py h2o | sh to submit them right away
- Remember to rerun grep or check.sh after these run!


## Preparing output

To minimize numerical noise, we want to work with relative energies. That is, we need to subtract the minimum energy from all of the other energies.

| halsio3.0004.0ut | 1CCSD (T)-F12b total energy | -757.368358155773 |
| :---: | :---: | :---: |
| halsio3,0005.out | !CCSD (T) -F12b total energy | -757.368299271901 |
| halsio3,0006.0u | !CCSD (T)-F12b total energy | -757.368349137881 |
| halsio3.0007.0ut | !CCSD (T)-F12b total energy | -757.368339420197 |
| halsio3,0008.out : | ICCSD (T) -F12b total energy | -757.368365731733 |
| halsio3,0009.0ut | !CCSD (T)-F12b total energy | -757.368338574281 |
| halsio3,0010.out : | !CCSD (T)-F12b total energy | -757.368241908086 |
| halsio3.0011.out : | ICCSD (T)-F12b total energy | -757.368363416914 |
| halsio3.0012.out : | !CCSD (T)-F12b total energy | -757.368280273263 |
| halsio3.0013.out: | !CCSD (T)-F12b total energy | -757.368293617617 |
| halsio3.0014.out : | ICCSD (T)-F12b total energy | -757.368370633874 |
| halsio3.0015.out : | ICCSD (T)-F12b total energy | -757.368372577147 |
| halsio3.0016.out: | !CCSD (T) -F12b total energy | -757.3683688118689 |

Figure: Want to go from this...

Use awk to do this, see relmaker.sh

## Anpass

With the relative energies in hand, we are ready to fit them to a function

```
!INPUT
TITLE
INDEPENDENT VARIABLES
DATA POINTS
233.-2
6F12.8,f20.12
    0.00000000
    0.02000000
    -0.0206060a
    -0.01560608
    0.01500000
    -0.01500006
    -0.01000000
    -0.01000000
    -0.01000000
    *-0.00000000 -0.01000000 0.00000000 
    0.01000000 0.00000000 0.00000000 -0.01000000 0.00000000 0.00000000
    0.01000000 
    * 0.01000000 
```



```
    -0.01060000 0.00000000 0.00000000 0.00000c00
    -0.01000000 
    -0.01000000 0.00000000
```



```
    -0.01000000 
    -0.01000000 0.01000000 c.0000000 
    -0.00560000 -0.01500000 0.00000000 
    -0.00500000 -0.01000000 
    -0.00500000 -0.00500000 -0.01000000 0.00000000 0.00000000 0.00000000
    -0.00560000 -0.00500000 0.00000000 -0.01000006 0.00000000 0.60800000
    -0.00560000 -0.00500000 0.60600000 0.00000606 -0.01000000 0.00006000
    -0.00500000}-0.00500000 0.00000000 0.00000000 0.00000000 -0.01600000
    -0.00500000 -0.00500000 0.00000000 0.00000000 0.00000000
    -0.00500000 -0.00500000 0.00000000 0.00000000 0.00000000 0.01000000
    -0.00500000 -0.00500000 0.00000000 0.00000000 0.01000000 0.00000000
    -0.00500000 -0.00500000 0.00000000 0.01000000 0.00000000
```

0.000600000000 0. 000175586858 0. 060100399569 a. 060698228397 0.000106381061 0.000059936736 0.000646525039 0.000073355273 0.000048164206 0.000683096066 0.000677338116
9. 06064336740 ?
0.000077338121
0. 060683096107
0.000048164204
0.060073355222
0.060073355222
0.000050488282
$0.0 е 0657912224$
0.000667912224
0.000053731551
0.000029200483
0.000044259147 0. 060019722496 0. 060054146554
0. 000048330215
0.000014855027
0.000048330227
0.000054146555
0.000019722511

Figure: Example anpass input file

## Anpass

## Formatting is again very important!



Figure: Example anpass input file

## Anpass

The first six columns in this case are the displacements. The final column is the relative energies, which you need to replace with your data

| ! INPUT |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TITLE |  |  |  |  |  |  |
| al.202 |  |  |  |  |  |  |
| INDEPENDENT VARIABLES |  |  |  |  |  |  |
| DATA POINIS |  |  |  |  |  |  |
| $233-2$ |  |  |  |  |  |  |
| (6F12.8, +20.12) |  |  |  |  |  |  |
| 0.00000000 | 0.00000000 | 0.000cอ000 | 0.00000000 | 0. 00000006 | 0.00000000 | 0.000006000000 |
| -0.02000000 | 0.00000000 | 0.000cocob | 0.00000000 | 0. 00000000 | 0.00000000 | 0. 000175586858 |
| -0.01500000 | -0.00500000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.000100399569 |
| -0.01500000 | 0.0000000 | 9.000¢0¢0¢ | 0.00000000 | 0.00000006 | 0.0000000 | 0. 000098228397 |
| -0.01500000 | 0.00500000 | 0.000c0000 | 0.00000060 | 0.00000000 | 0.00000000 | 0.000106381061 |
| -0.01000000 | -0.01000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0,000059936736 |
| -0.01000000 | -0.00500000 | 0.000c0e00 | 0.00000000 | 0.00000000 | 0.00000000 | 0.000046525039 |
| -0.01000000 | 0.00600080 | -0.01060000 | 0.00000000 | 0, 80000000 | 0.00000000 | 0.000073355273 |
| -0.01060000 | 0.00600060 | D. 000 COCOB | -0.01000000 | 0. 00000006 | 0.00000000 | 0.000048164206 |
| -0.01000000 | 0.0000000 | 0. 00000000 | 0.00000000 | -0.01000000 | 0.00000000 | 0. 000083096066 |
| -0.01000000 | 0.0000000 | 0.000¢0000 | 0.00000000 | 0.00000000 | -0.01000000 | 0.000077338116 |
| -0.01000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0. 00000000 | 0.0000000 | 0.000043367402 |
| -0.01000000 | 0.0000000 | 0. 0 00c0000 | 0.00000000 | 0. 0000000 | 0.01000000 | 0.000077338121 |
| -0.01000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.01000000 | 0.00000000 | 0.000083096107 |
| -0.01000000 | 0.00000000 | 0.00000000 | 0.01000000 | 0, 00000000 | 0.00000000 | 0. 000048164204 |
| -0.01000000 | 0.00008080 | 0.010c0cob | 0.00000000 | 0. 00000006 | 0.00000000 | 0.000073355222 |
| -0.01060000 | 0.0050000 | -.00060¢0¢ | 0.00000000 | 0. 00000006 | 0.00000000 | 0. 000050488282 |
| -0.01000000 | 0.01600000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0. 000067912224 |
| -0.00500000 | -0.01500000 | 0.00000000 | 0.00000000 | 0.00000006 | 0.00000000 | 0. 000053731551 |
| -0.00560000 | -0.01000000 | 0.00000¢00 | 0.00000000 | 0.0000000¢ | 0.00000000 | 0.000029200483 |
| -0.00500000 | -0.00500000 | -0.01000000 | 0.00000000 | 0.00009000 | 0.00000000 | 0, 000044259147 |
| -0.00500000 | -0.00500000 | 0.00060000 | -0.01000000 | 0.00000000 | 0.00000000 | 0.000019722496 |
| -0.06500000 | -0.00500000 | 0.000c0000 | 0.00000000 | -0.01000000 | 0.00000000 | 0.000054146554 |
| -0.06580000 | -0.00500000 | อ. 000 COCOQ | 0.00000000 | 0. 00000000 | -0.01000000 | 0.000048330219 |
| -0.06560000 | -0.00500000 | 0. 00000000 | 0.00000000 | 0. 00000000 | 0.00000000 | 0.000014855027 |
| -0.00500000 | -0.00500060 | 0.00000000 | 0.00000000 | 0, 00000000 | 0.01000000 | 0.000048330227 |
| -0.00500000 | -0.00500000 | 0.00000000 | 0.00000000 | 0.01000000 | 0.0000000 | 0.000054146555 |
| -0.00500000 | -0.00500000 | 0.000¢0000 | 0.01000000 | 0. 0000000 | 0.0000000 | 0, 000019722511 |
| -0.00500000 | -0.00500000 | 0.010c0000 | 0.0popooco | 0.00000000 | 0.00000000 | 0.000044259223. |

## Anpass

Once the input file is set up:

- Run anpass with $\$$ anpass < anpass1.in > anpass1.out
- Notice the 1 here, this is the first of two Anpass runs
- Go to the bottom of the first output file and make sure the sum of squared residuals is less than about $10^{-16}$



## Anpass

After this:

- Copy anpass1.in to anpass2.in
- Copy the "long line" from anpass1.out to the bottom of anpass1.in, above END OF DATA
- Cut the !STATIONARY POINT line and paste it above the "long line"



## Anpass



Figure: anpass1.in bottom

## Anpass



Figure: anpass2.in bottom

## Anpass

What is the long line?

- In fitting our function, anpass can find a new minimum that is slightly different from the one from Molpro
- The long line is the series of displacements that correspond to this minimum, along with its energy
- The "!" is a comment, so we are telling anpass that the long line is a stationary point and to run a slightly different calculation


## Anpass

The other important thing anpass gives us are the force constants in fort. 9903

| 53 |  |  |  |  |
| :--- | :--- | :--- | :--- | ---: |
| 0 | 0 | 0 | 0 | 0.000000000288 |
| 6 | 6 | 0 | 0 | 2.879641150306 |
| 6 | 6 | 6 | 6 | 3.268664626615 |
| 5 | 5 | 0 | 0 | 3.382416943673 |
| 6 | 6 | 5 | 5 | 16.919503402043 |
| 5 | 5 | 5 | 5 | 7.72004961100 |
| 4 | 4 | 0 | 0 | 0.432161118353 |
| 6 | 6 | 4 | 4 | -3.489227624058 |
| 5 | 5 | 4 | 4 | -4.247995562121 |
| 4 | 4 | 4 | 4 | 2.317438189761 |
| 6 | 5 | 3 | 0 | -9.100591142294 |
| 6 | 5 | 4 | 3 | -0.014823366425 |
| 3 | 3 | 0 | 0 | 2.5142802226 |
| 6 | 6 | 3 | 3 | 22.039113667990 |
| 5 | 5 | 3 | 3 | 21.784976166893 |

Figure: fort. 9903
This takes us back into intder

## Intder

intder_geom.in

Anpass gave us a new equilibrium geometry in terms of symmetry internal coordinate displacements. We need to use Intder to turn that into a Cartesian geometry. Intder's main purpose is undertaking these transformations for us, hence calling it the glue

- Copy your intder.in file from pts and call it intder_geom.in
- Change the number of displacements to 1 and delete all of the displacements
- You may want to keep at least one or two for alignment reference
- Take the nonzero lines from under WHERE ENERGY IS AT in anpass1.out to be the new displacements


## Intder

intder_geom.in
I'm mixing my examples here unfortunately, so pay attention to the displacements section only


Figure: intder.in from pts


Figure: intder_geom.in

These are the same numbers from the long line, but formatted more easily for our use

## Intder

intder.in
intder_geom will produce a new equilibrium geometry in Cartesian coordinates that we can stick back in to a slightly different type of intder file

- Copy any intder.in to the freqs directory
- Take the new geometry from the very bottom of intder_geom.out and replace the one in intder.in
- Delete everything below the Cartesian coordinates
- The next line needs to be the atoms, with their masses
- You almost definitely want to copy one of these lines from another file to get the alignment right


## Intder

intder.in


Figure: Our intder.in file so far

## Intder

intder.in

Now we need to read in the force constants from fort. 9903

- You can read fort. 9903 in directly with :r fort.9903, but there is a lot of processing to do
- Instead, use format.sh; there are two ways to use it:
- My preferred way is to run it from inside vim with :r !format.sh
- But you can also run it at the command line \$ format.sh > intder.bot
and save the output into a file that you then read in to intder.in


## Intder

intder.in


Figure: Part of the final intder.in file

## Intder

intder.out

- Run intder: \$ intder < intder.in > intder.out
- Verify that the harmonic frequencies at the very bottom correspond to those from Molpro (if those have finished)
- Run \$ tennis.sh to move the intder output files to the filenames expected by spectro



## Spectro <br> spectro.in

You should be given a spectro.in file looking something like this


## Spectro <br> spectro.in

## Don't worry about these, probably



Input directives, see manual

## Spectro <br> spectro.in

You will definitely need to change these


## Spectro

spectro.in
Change these too, from nist.gov/pml/atomic-weights-and-isotopic-compositions-relative-atomic-masses


## Spectro

spectro.in
You shouldn't need to touch these, they should resemble the simple internals from intder


## Curvilinear coordinates

## Spectro <br> spectro.in

## We'll talk more about these in a minute



Resonances

## Spectro <br> spectro.out

- Run spectro with
\$ spectro < spectro.in > spectro.out
- Open spectro.out and search for "FUND" and it should look something like below



## Spectro <br> spectro.out

- Now, jump to the top and search for "FERMI"
- This will take you to the resonances
- We only worry about the Fermi and Coriolis ones



## What is a resonance?

## Fermi resonance

If two vibrational modes have the same symmetry and similar energies, the higher energy mode will shift to higher energy and the lower to lower energy, basically to get away from each other


## What is a resonance?

The definition may be slightly different for Coriolis resonances, but there wasn't a Wikipedia page

## Fermi type 1

An overtone band, an integer multiple of one frequency, has a frequency similar to that of another band.

We see an example of this in our spectro.out file. $2 \nu_{6} \approx \nu_{5}$

| FERMI RESONANCE |  |  |  |  | TYPE $1 \quad 2 \mathrm{WI}=\mathrm{WJ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | I | J | WI | WJ | DIFF | PHI IIJ | EST. PERTURB. |
| X | 6 | 6 | 5 | 301.99 | 524.33 | 79.66 | -11.380 | 0.000 |

$80 \mathrm{~cm}^{-1}$ is a pretty huge difference, so the estimated perturbation is zero, but often these frequencies line up perfectly and have a substantial effect

## What is a resonance?

## Fermi type 2

The combination of two lower frequency modes have a frequency similar to one higher frequency mode.

> Example
> Let $\nu_{6}=234.5, \nu_{5}=364.3$, and $\nu_{2}=600.1 \mathrm{~cm}^{-1}$. Then $\nu_{6}+\nu_{5}=598.8 \approx \nu_{2}$, and we could expect a larger perturbation than seen for our type 1 example.

## Spectro

spectro.in again

How do we account for these resonances? We already saw how to handle type I resonances in the example spectro file


This says that we have one type 1 Fermi resonance, in which $2 \nu_{6}=\nu_{5}$.

## Spectro

spectro.in again

To handle a type 2 Fermi resonance, such as the one in our example, we would add another block like this:

## \# FERMI2 \#\#\#\#\#\# <br> 1 <br> $6 \quad 5 \quad 2$

which again says that $\nu_{6}+\nu_{5}=\nu_{2}$

## Spectro

spectro.in again

## Fermi polyad

We can also have a situation where multiple combinations come close to a single frequency. For example, both $\nu_{6}+\nu_{5}$ and $2 \nu_{4}$ could be close to $\nu_{2}$. This results in a Fermi polyad, which should be handled separately.


Figure: Handling a Fermi polyad for $\mathrm{Ti}_{2} \mathrm{O}_{2}$

## Spectro

spectro.in again

Basically if you have a polyad you need to make one of those RESIN arrays. Everything on the right hand side of the equation gets a row with a one in its column, and everything on the left hand side gets a row. In the previous example, we had a row
1
0
0
0
0
0
which represents $\nu_{1}$, which appeared in two type 1 Fermi resonances, $2 \nu_{4}=\nu_{1}$ and $2 \nu_{5}=\nu_{1}$. The corresponding row for $\nu_{5}$ is

$$
\begin{array}{llllll}
0 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

## Spectro

spectro.in again

The combinations of modes that make $\nu_{5}$ and $\nu_{1}$ get their own rows. Since in this case they are type 1 Fermi resonances, there are twos in those positions. For $2 \nu_{4}$, which equals $\nu_{1}$, we get the row
0
0
0
2
0
0
and so on. If we had a type 2 Fermi resonance, say $\nu_{3}+\nu_{4}=\nu_{1}$ too, we would add a row that looks like
$\begin{array}{llllll}0 & 0 & 1 & 1 & 0 & 0\end{array}$

The ordering of these rows does not matter, just make sure that every left and right hand side is accounted for in equations like $\nu_{3}+\nu_{4}=\nu_{1}$.

## Spectro

spectro.in again

## Coriolis resonances

I'm not as sure about the physical meaning of these, but think of them the same way. Some combination of frequencies bumps into each other and wants to push the other away. Regardless, we need to handle them.
***********************************************************
CORIOLIS RESONANCES WI = WJ
***********************************************************

|  | I | J | WI | WJ | DIFF | ZETA | DeltaALPHA |
| :--- | :--- | :--- | :---: | :---: | ---: | ---: | :--- |
| ALPHA | 3C | 2 | 679.87 | 774.88 | -95.01 | -1.000 | -0.00458 |
| ALPHA | 4C | 1 | 647.70 | 805.41 | -157.71 | -0.351 | -0.00034 |
| ALPHA | 5C | 4 | 524.33 | 647.70 | -123.37 | -0.936 | -0.00311 |

Figure: Coriolis resonances in spectro.out

## Spectro

spectro.in again

The way I read these are the first number, then the second, then the letter. So $3,2, C$ in the case of the first one here.

|  | +10 | CORIOLIS RESONANCES |  |  | $\mathrm{WI}=\mathrm{WJ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | ***** |
|  | I | J | WI | WJ | DIFF | ZETA | DeltaALPHA |
| ALPHA | 3C | 2 | 679.87 | 774.88 | -95.01 | -1.000 | -0.00458 |
| ALPHA | 4C | 1 | 647.70 | 805.41 | -157.71 | -0.351 | -0.00034 |
| ALPHA | 5C | 4 | 524.33 | 647.70 | -123.37 | -0.936 | -0.00311 |

Figure: Coriolis resonances in spectro.out

## Spectro

spectro.in again
And that's basically the way you put them in. The weirdest thing is that the letters are given by a vector. $\mathrm{A} \rightarrow 100, \mathrm{~B} \rightarrow 010$, and $\mathrm{C} \rightarrow 001$


Figure: Coriolis resonances in spectro.in

## Spectro

## Vibrational data

- Run spectro again to finish up
- You can check FUND again to make sure it still looks right, but to see where the resonances are taken into account, we need to search instead for ZPT
- Here, look for the "STATE NO." corresponding to 1 plus whatever mode. State 7 for $\nu_{6}$, for example.
- You can also use my script inspectro, which will print the harmonic, anharmonic, and resonance corrected frequencies, although it sometimes prints a lot more too
- You also want State 1, which is the ZPT. We report this as zero-point vibrational energy (ZPVE)


## Spectro

Vibrational data


Figure: Sample ZPT section

| ZPT $=$ |  |  |  |
| ---: | ---: | ---: | ---: |
| 1860.3 |  |  |  |
| HARM | FUND | CORR |  |
| 805.4 | 794.6 | 794.6 |  |
| 774.9 | 762.6 | 762.6 |  |
| 679.9 | 667.2 | 667.2 |  |
| 647.7 | 634.9 | 634.9 |  |
|  | 524.3 | 520.8 | 520.7 |
|  | 302.0 | 300.4 | 300.4 |

Figure: Sample inspectro output

## Spectro

Rotational data

Spectro also outputs rotational constants, which can be found by searching:

- BZA - $A, B$, and $C$ rotational constants for every vibrational state. BZA is usually $A, B X A$ is $B$, and BYA is $C$. In this case we have $A_{0}, B_{0}$, and $C_{0}$.


## VIBRATIONAL STATE NON-DEG(Vt) : 0

$$
\begin{equation*}
0 \tag{0}
\end{equation*}
$$

$$
0
$$

$$
0
$$

$$
0
$$

## ROTATIONAL CONSTANTS (CM-1)

$$
\begin{array}{cc}
\text { BXA } & \text { BYA } \\
0.2148329 & 0.1295869
\end{array}
$$

BZA
0.3275038

## Spectro

Rotational data

Spectro also outputs rotational constants, which can be found by searching:

- DELTA, quartic distortion constants

\[

\]

## MHz

0.0021030914
0.0082114140
$-0.0014293852$
0.0008129388
0.0024252037

## Spectro

Rotational data

Spectro also outputs rotational constants, which can be found by searching:

- PHI, sextic distortion constants

IN CM-1
PHI J :
PHI K :
PHI JK :
PHI KJ :
phi j :
phi jk:
phi k :
$0.6628031116 \mathrm{D}-13$
$0.1468897092 \mathrm{D}-11$
$-0.2289305553 \mathrm{D}-12$
-0.7127345498D-12
$0.3301218541 \mathrm{D}-13$
$-0.3351971470 \mathrm{D}-13$
$0.6056513421 \mathrm{D}-12$

IN Hz
0.1987033740D-02
$0.4403642696 \mathrm{D}-01$
-0.6863165389D-02
-0.2136724426D-01
$0.9896804209 \mathrm{D}-03$
-0.1004895766D-02
0.1815697045D-01

## Reporting

## Units and precision

- Vibrational frequencies to 1 decimal place in $\mathrm{cm}^{-1}$, ex. 627.4 $\mathrm{cm}^{-1}$
- Harmonic frequencies are denoted $\omega_{n}$ (omega)
- Anharmonic frequencies are denoted $\nu_{n}$ (nu)
- $A, B$, and $C$, rotational constants to 1 decimal place in MHz
- $\Delta$ and $\Phi$ constants to 3 decimal place but no more than 5 sig figs. Adjust units accordingly. For example, instead of 35154.123 kHz , use 35.154 MHz
- Bond lengths to 5 decimal places
- Bond angles (I think) to 3 decimal places


## Reporting

Mode descriptions

Given symmetry internal coordinates, $S_{n}$, like those on the left, which go in the Computational Details, we often want to describe the resulting vibrational modes as linear combinations of them, as on the right

$$
\begin{array}{r}
S_{1}\left(a_{1}\right)=r\left(H_{1}-C_{1}\right) \\
S_{2}\left(a_{1}\right)=r\left(H_{1}-C_{2}\right) \\
S_{3}\left(a_{1}\right)=\frac{1}{\sqrt{2}}\left[r\left(C_{2}-C_{3}\right)+r\left(C_{2}-C_{4}\right)\right] \\
S_{4}\left(a_{1}\right)=\frac{1}{\sqrt{2}}\left[r\left(C_{3}-H_{2}\right)+r\left(C_{4}-H_{3}\right)\right] \tag{4}
\end{array}
$$

| Mode | Description | Symmetry |
| :--- | :--- | :--- |
| $\omega_{1}$ | $0.96 S_{1}$ | $a_{1}$ |
| $\omega_{2}$ | $0.95 S_{4}$ | $a_{1}$ |
| $\omega_{3}$ | $1.00 S_{8}$ | $b_{2}$ |
| $\omega_{4}$ | $0.68 S_{2}+0.28 S_{3}$ | $a_{1}$ |
| $\omega_{5}$ | $0.74 S_{5}+0.24 S_{2}$ | $a_{1}$ |
| $\omega_{6}$ | $0.90 S_{10}+0.12 S_{9}$ | $b_{2}$ |
| $\omega_{7}$ | $0.58 S_{11}+0.31 S_{9}+0.09 S_{10}$ | $b_{2}$ |

## Reporting

Mode descriptions
Fortunately, you can find this information by searching "ASSIGNMENT" in intder.out

```
VIBRATIONAL ASSIGNMENTS
    MODE CM-1
    DOMINANT COMPONENTS OF TED
\begin{tabular}{lllll}
1 & 302.0 & \(4(100.0)\) & & \\
2 & 524.3 & \(2(95.2)\) & \(-1(\) & \(4.8)\) \\
3 & 647.7 & \(3(100.0)\) & & \\
4 & 679.9 & \(6(100.0)\) & & \\
5 & 774.9 & \(5(100.0)\) & & \\
6 & 805.4 & \(1(95.2)\) & \(2(\) & \(4.8)\)
\end{tabular}
```

In this case, basically every frequency lines up $1: 1$ with a symmetry internal coordinate, but as shown on the previous slide this is not always the case


[^0]:    

